Chapter 10 – Fluids

Up to now we only considered rigid objects, now we look at deformable materials

The three common **phases of matter** are solid, liquid, and gas.

A **solid** has a definite shape and size (not easily compressed).

A **liquid** has a fixed volume but can be any shape (not easily compressed).

A **gas** can be any shape and expands to fill its container. It can be easily compressed.

Liquids and gases both flow, and are called **fluids.**

The division of matter into three phases is not always simple.
What is butter?
Fourth phase – **plasma** – at very high temperatures, ionized atoms

Density and Specific Gravity

Iron is not necessarily "heavier" than wood, but it is more **dense.**

The **density** ρ of an object is its mass per unit volume:

$$\rho = \frac{m}{V}$$

The SI unit for density is kg/m^3. Density is also sometimes given in g/cm^3; to convert g/cm^3 to kg/m^3, multiply by 1000.

Ex. 10-1 What is the mass of a solid iron ball of radius 18 cm?

$$V = \frac{4}{3}\pi r^3 \qquad \rho = 7.8\times 10^3\, kg/m^3$$

m=190 kg

<u>**Specific Gravity:**</u> is the ratio of the density of a substance to the density of water at 4 Degrees Celsius

Water at 4°C has a density of 1 g/cm^3 = 1000 kg/m^3.

TABLE 10–1
Densities of Substances[†]

Substance	Density, ρ (kg/m^3)
Solids	
Aluminum	2.70×10^3
Iron and steel	7.8×10^3
Copper	8.9×10^3
Lead	11.3×10^3
Gold	19.3×10^3
Concrete	2.3×10^3
Granite	2.7×10^3
Wood (typical)	$0.3 - 0.9 \times 10^3$
Glass, common	$2.4 - 2.8 \times 10^3$
Ice (H_2O)	0.917×10^3
Bone	$1.7 - 2.0 \times 10^3$
Liquids	
Water (4°C)	1.00×10^3
Blood, plasma	1.03×10^3
Blood, whole	1.05×10^3
Sea water	1.025×10^3
Mercury	13.6×10^3
Alcohol, ethyl	0.79×10^3
Gasoline	0.68×10^3
Gases	
Air	1.29
Helium	0.179
Carbon dioxide	1.98
Water (steam) (100°C)	0.598

[†] Densities are given at 0°C and 1 atm pressure unless otherwise specified.

$$\rho = \frac{m}{V}$$

Temperature and pressure affect the density of substance.

- Compare **iron and wood**
- Notice how dense **mercury** is
- Note the low density of **gases**
- **Specific density** - dimensionless

Pressure in Fluids

Pressure is defined as the force per unit area.

$$\boxed{P = \frac{F}{A}}$$

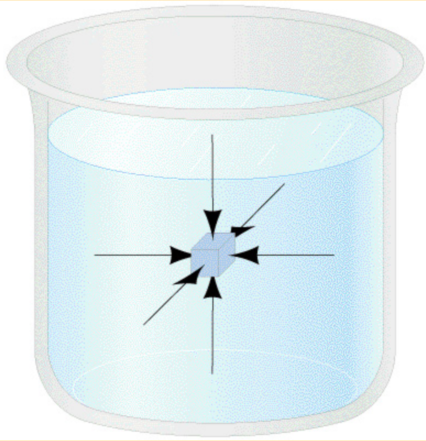
Force is a vector   BUT

Pressure is a SCALAR; the unit of pressure in the SI system is pascal (Pa):   1 Pa = 1 N/m^2

Ex. 10-2 The two feet of a 60-kg person cover an area of 500 cm^2.
(a) Determine the pressure exerted by the two feet on the ground.
(b) If the person stands on one foot, what is the pressure?

$(a) P = 12\times 10^3\, N/m^2 \quad (b) P = 24\times 10^3\, N/m^2$

Fluids exert pressure in all directions



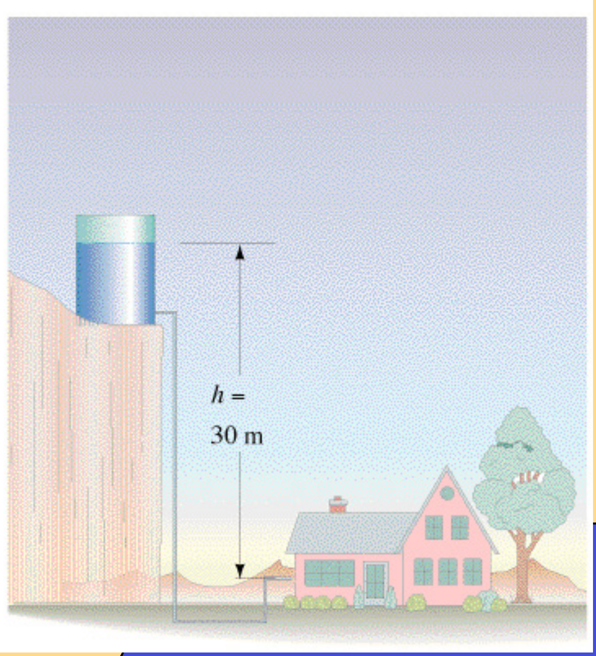
1. Pressure is the **same in every direction** in a fluid at a given depth; if it were not, the fluid would flow.
2. Force from fluid pressure acting on any object immersed in the fluid is applied **perpendicular** to the object's surface. If there were parallel component on surface, the surface would react and apply a force on the fluid, which would flow.

Pressure in Fluids

The pressure at a depth *h* below the surface of the liquid is due to the **weight** of the liquid above it.

$$F = mg = \rho V g = \rho A h g \Rightarrow \boxed{P = \frac{F}{A} = \frac{\rho A h g}{A} = \rho g h} \quad \boxed{\Rightarrow \Delta P = \rho g \Delta h}$$

The relation does not depend on the area.
This relation is valid for any liquid whose ***density is constant*** and does not change with depth = ***incompressible*** fluid.



The pressure at equal depths within a uniform liquid is the same

Ex. 10-3 The surface of the water in a storage tank is 30 m above the water faucet in the kitchen of a house. Calculate the difference in water pressure between the faucet and the surface of the water in the tank

$$\Delta P = 2.9 \times 10^5 \, N/m^2$$

To solve the exercise above we assumed that water is incompressible, but we know that at great depths in the ocean r increases by compression.

Atmospheric Pressure

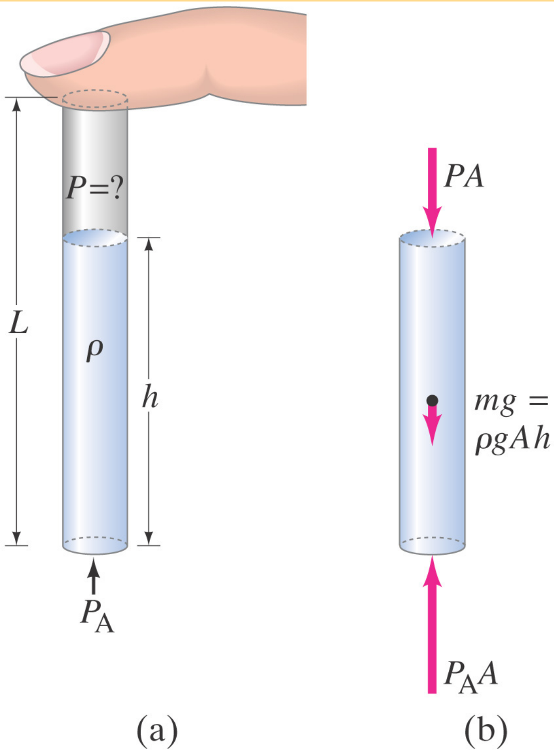
Pressure of the Earth's atmosphere changes with depth
r varies with altitude and weather

At sea level the atmospheric pressure is about

$$1.013 \times 10^5\ \mathrm{N/m^2}$$

this is called one atmosphere (atm).

This pressure does not crush us, because just like balloons, our cells maintain an internal pressure that balances it.



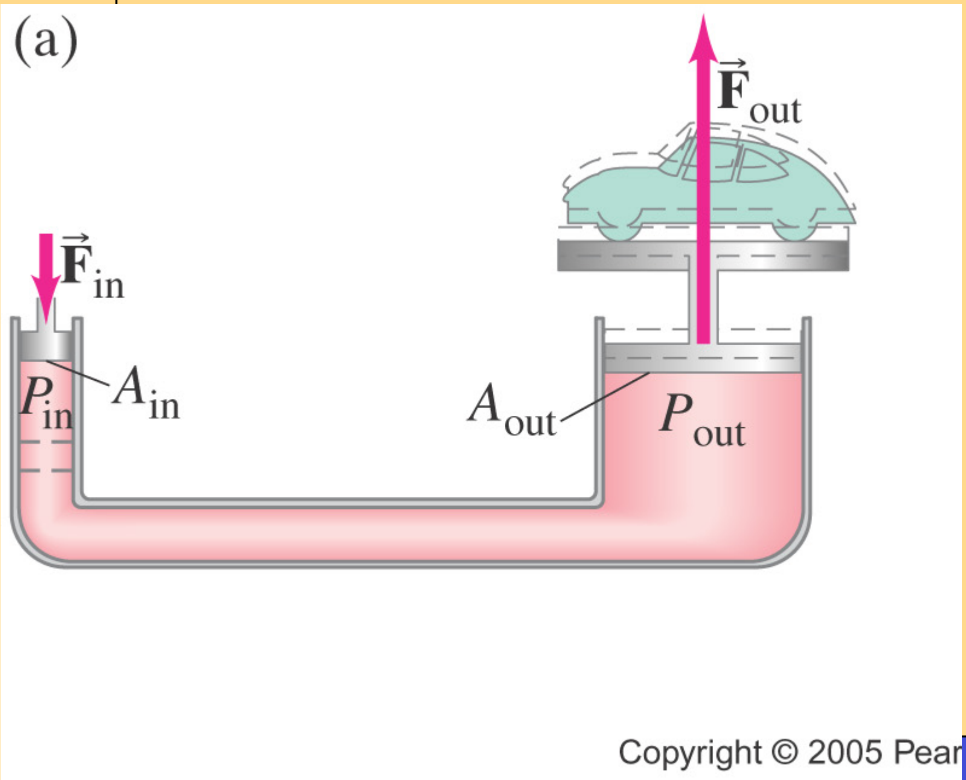
Ex. 10-4 Finger holds water in a straw

Air pressure inside the top of the straw has to be smaller than the atmospheric pressure outside the straw

Pascal's Principle

Pascal's Principle: If an external pressure is applied to a confined fluid, the pressure at every point within the fluid increases by that amount.

This principle is used, for example, in hydraulic lifts



Input and output pistons at the same height:

$$P_{out} = P_{in}$$

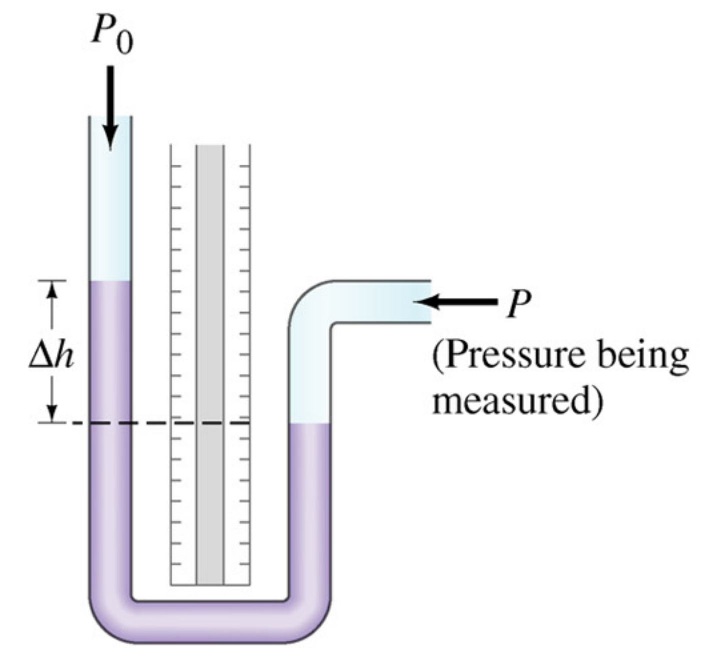
$$\frac{F_{out}}{A_{out}} = \frac{F_{in}}{A_{in}} \Rightarrow F_{out} = F_{in}\frac{A_{out}}{A_{in}}$$

Measurement of Pressure

Most pressure gauges measure the pressure above the atmospheric pressure – this is called the gauge pressure.

The absolute pressure is the sum of the atmospheric pressure and the gauge pressure.

$$P = P_A + P_G$$



(a) Open-tube manometer

Manometer. The pressure in the open end is atmospheric pressure; the pressure being measured causes the fluid to

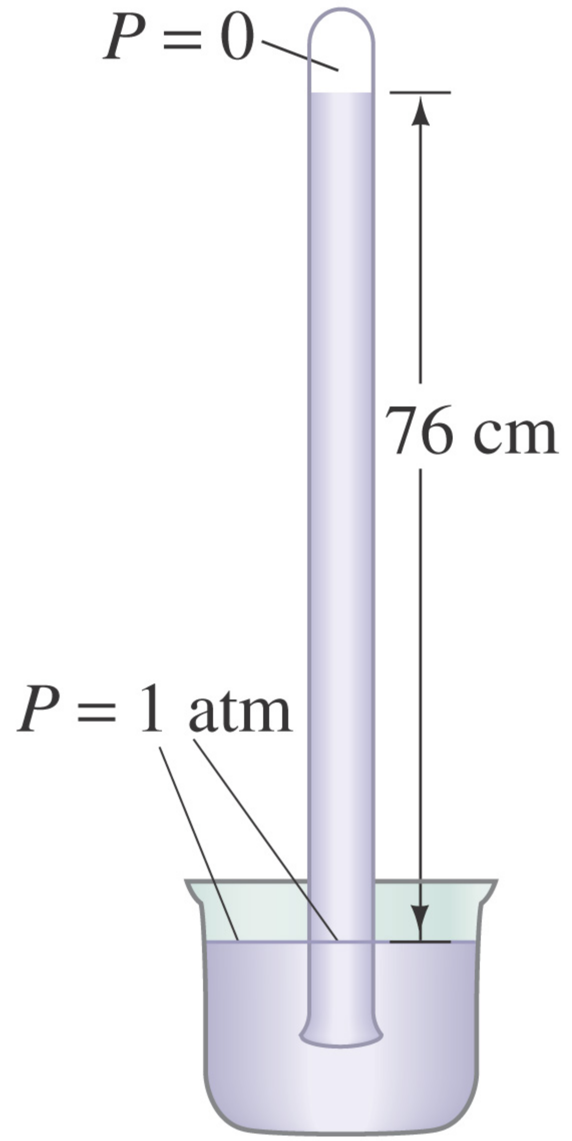
rise, if $P>P_0$

or fall, if $P<P_0$

Gauge pressure

$$P = P_0 + \rho g \Delta h$$

Barometer



This is a mercury barometer, developed by Torricelli to measure atmospheric pressure. The height of the column of mercury is such that the pressure in the tube at the surface level is 1 atm.

Height=76cm

$$P = \rho g \Delta h = (13.6\times 10^3)(9.8)(0.760) =$$

$$= 1.013\times 10^5\, N/m^2 = 1 atm$$

Any liquid can serve in a Torricelli-style barometer, but the most dense ones are the most convenient.

For example, a column of water would be 10.3 m high.

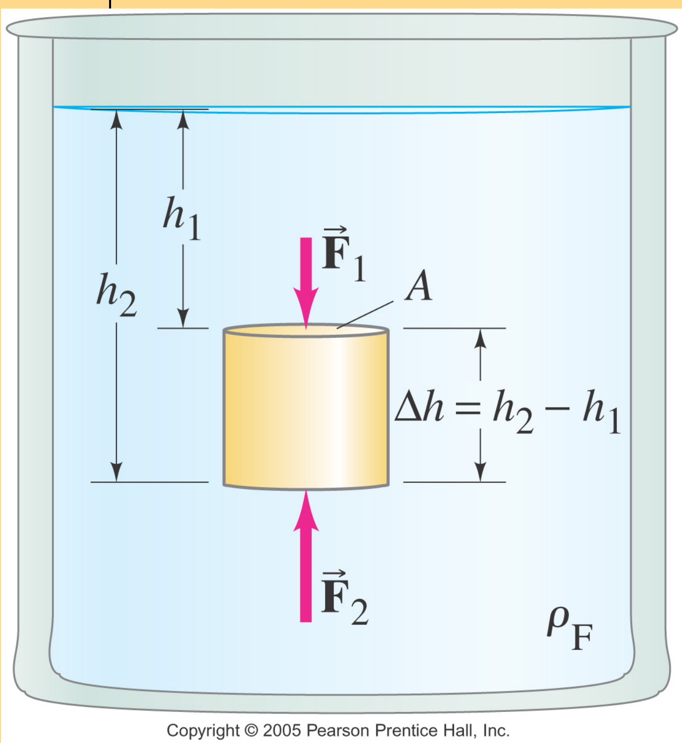
Buoyancy

Examples of **buoyancy:**

- Objects submerged in a fluid appear to weigh less than outside
- Some objects float

Besides the force of gravity acting downwards, there is the **buoyant force** exerted by the liquid upward.

The buoyant force occurs because the pressure in a fluid increases with depth



$$\begin{aligned} F_B = F_2 - F_1 &= \rho_F g A (h_2 - h_1) \\ &= \rho_F g A \,\Delta h \\ &= \rho_F V g \\ &= m_F g, \end{aligned}$$

Buoyant force on the object is equal to the weight of fluid displaced by the object

r_F is the density of the fluid NOT the object.
The pressure/force is exerted by the fluid

Archimedes' s Principle

<u>Archimede' s Principle:</u>

Buoyant force on the object is equal to the weight of fluid displaced by the object

$$\boxed{F_B = m_F g}$$

The volume of liquid displaced = the volume of the submerged object.

Ex. 10-6 Consider two identical buckets of water filled to the brim. One bucket contains only water, the other has a piece of wood floating in it. Which bucket has the greater weight (if any)?

Both buckets weigh the same. The piece of wood displaces a volume of water with weight equal to the weight of the wood.

Ex. 10-7 A 70-kg statue lies at the bottom of the sea. Its volume is 3.0x10^4 cm^3. How much force is needed to lift it?
(for seawater: r=1.025x10^3 kg/m^3)

F needed is 390 N

EUREKA

Eureka (Greek "I have found it") is an exclamation used as an interjection to celebrate a discovery. It is most famously attributed to Archimedes; he reportedly uttered the word when, while bathing, he suddenly understood that ***the volume of an irregular object could be calculated by finding the volume of water displaced when the object was submerged in water.*** After making this discovery, he is said to have leapt out of his bathtub and run through the streets of Syracuse naked.

Archimedes' insight led to the solution of a problem that had been asked of him by Hiero of Syracuse, on how to assess the purity of an irregular golden crown: by dividing the object's weight by its volume, one could calculate its density, an important indicator of purity.

Gold or Lead?

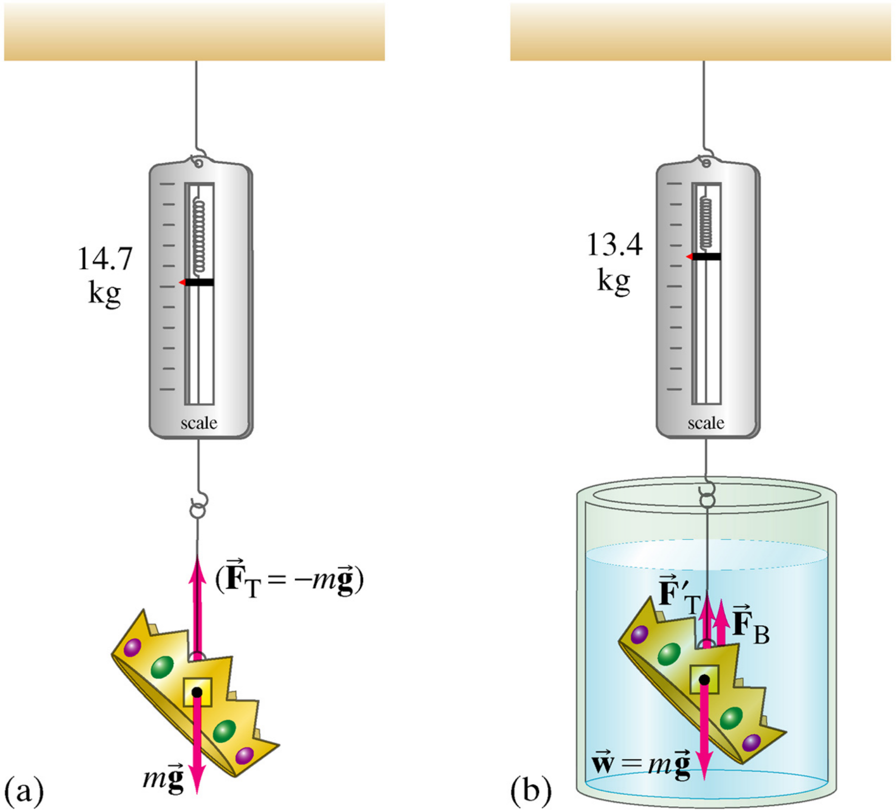
Ex. 10-8 When a crown of mass 14.7 kg is submerged in water, an accurate scale reads only 13.4 kg. Is the crown of gold?

$$w' = w - F_B \Rightarrow w - w' = F_B = \rho_F V g$$

$$w = mg = \rho_o V g$$

$$\frac{w}{w - w'} = \frac{\rho_o V_o g}{\rho_F V_F g} = \frac{\rho_o}{\rho_F}$$

$$\rho_o = 11.3 \times 10^3\, kg/m^3 \,(lead)$$

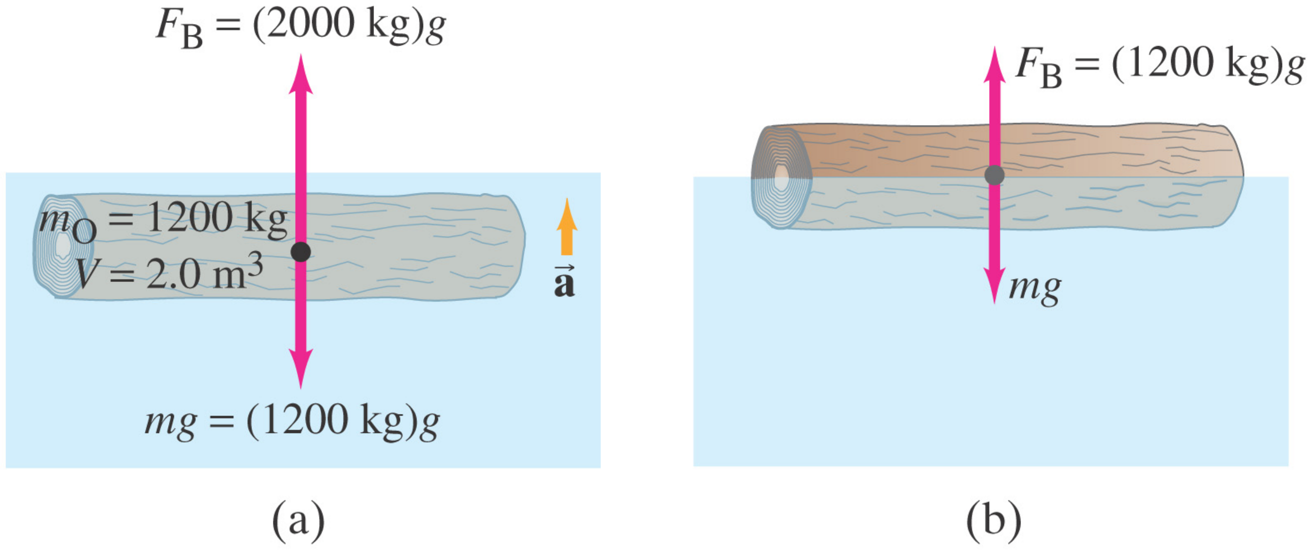


Archimedes' s Principle (float)

An object floats on a fluid if its density is less than that of the fluid.

If the object' s density is less than that of water, there will be an upward net force on it, and it will rise until it is partially out of the water.

(a)



(b)

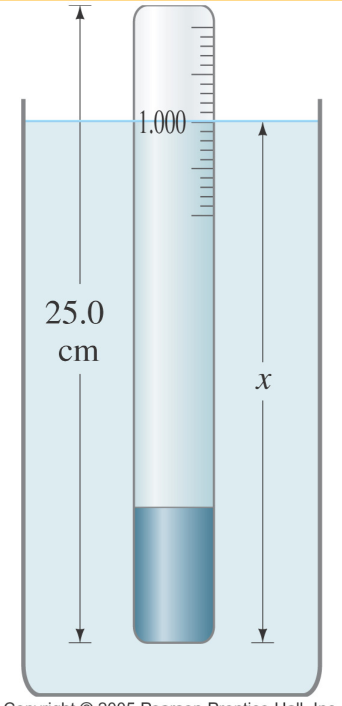
$$F_B = m_F g = \rho_F V g = (2.0\times10^3)g$$

$$\rho_F = 1.00\times10^3 kg/m^3$$

$$For\ equilibrium:\ F_B = m_O g$$

$$\rho_F V_{displ} g = \rho_O V_O g \Rightarrow \boxed{V_{displ} = V_O \frac{\rho_O}{\rho_F}}$$

Exercises



Ex. 10-9 Suppose a 25.0-cm tube of mass 45.0 g and cross-sectional area 2.00 cm^2 in water. In equilibrium, how deep will it be submerged?

$$V_{displ} = V_O \frac{\rho_O}{\rho_F}$$

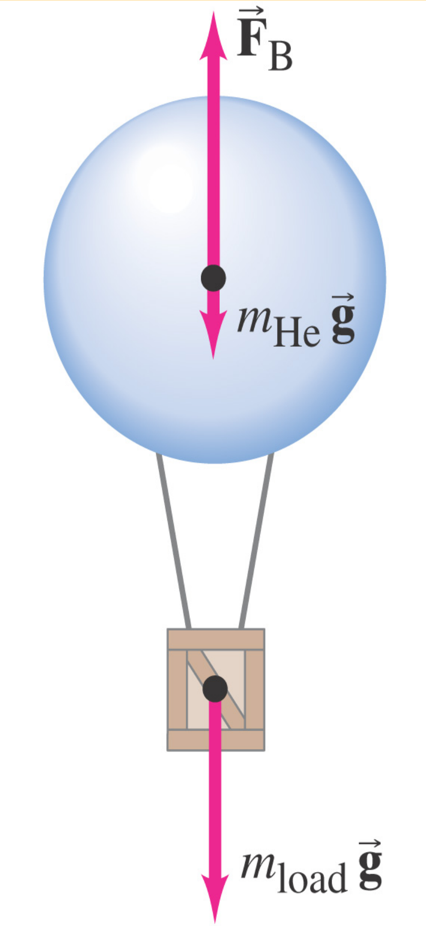
$$Ax = A(25.0)\frac{\rho_O}{\rho_F} \Rightarrow x = 22.5cm$$

Air is a fluid and it too exerts a buoyant force

Ex. 10-10 What volume V of helium is needed if a balloon is to lift a load of 180 kg (including the weight of the balloon)?

$$F_B = (m_{He} + 180kg)g$$

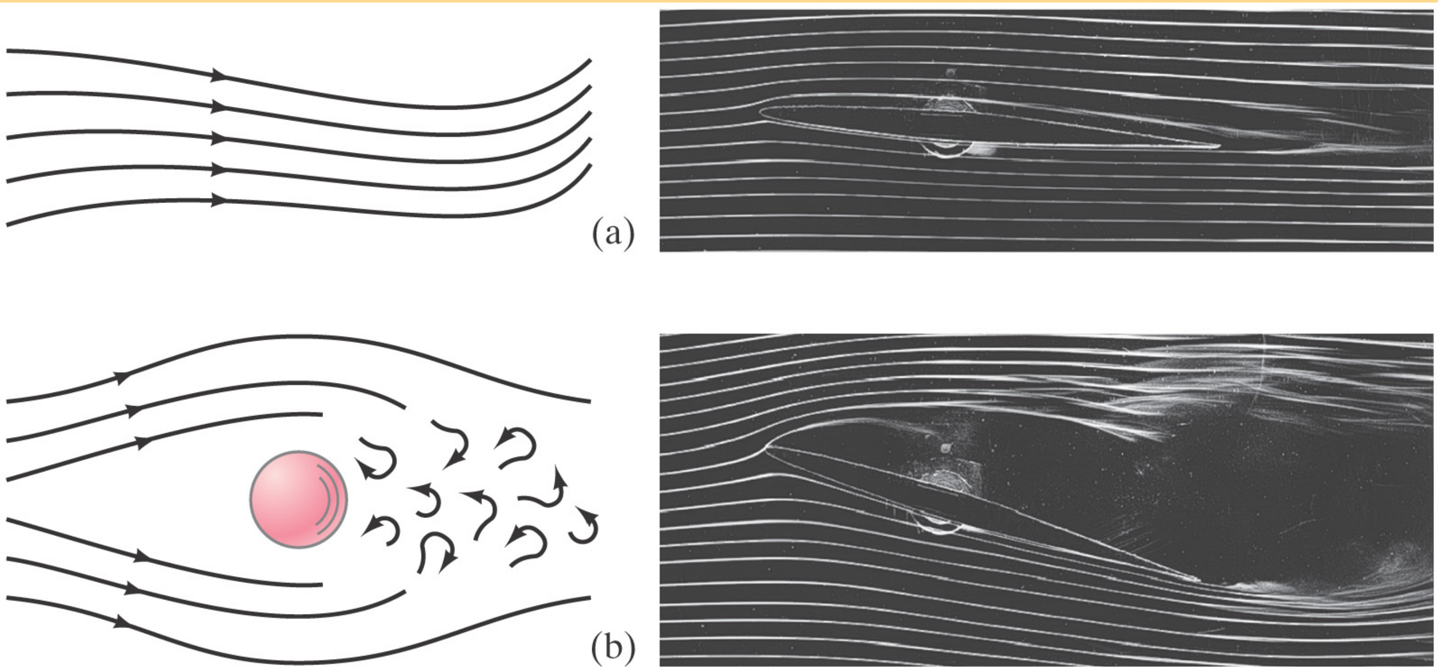
$$\rho_{air} Vg = (\rho_{He} V + 180kg)g \Rightarrow V = 160m^3$$



Fluids in Motion – Fluid Dynamics

If the flow of a fluid is smooth, it is called **streamline** or **laminar** flow (a).

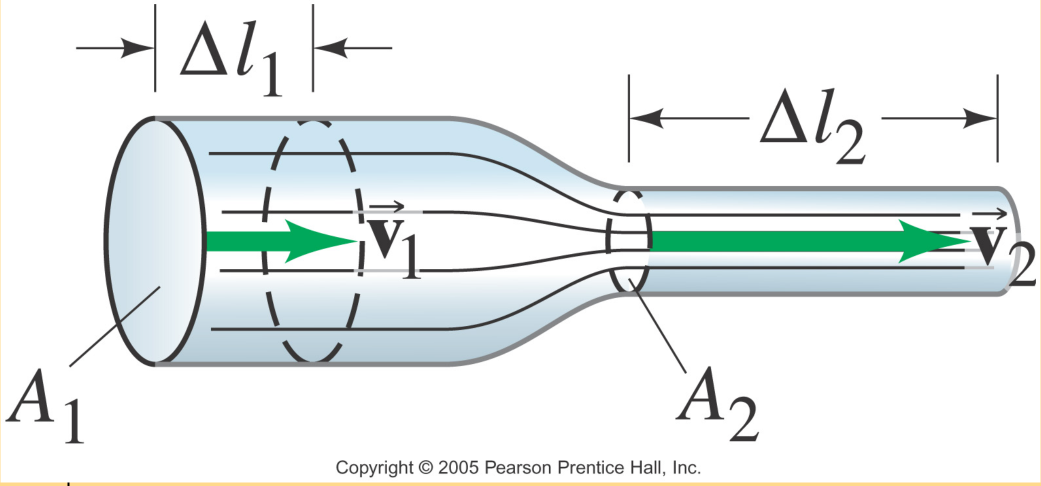
Above a certain speed, the flow becomes **turbulent** (b). Turbulent flow has small whirlpool-like circle - eddies; the **viscosity** of the fluid is much greater when eddies are present.



Laminar Flow

We will deal with laminar flow.

The **mass flow rate** is the mass that passes a given point per unit time. The flow rates at any two points must be equal, as long as no fluid is being added or taken away.



$$mass\ \ flow\ rate: \frac{\Delta m}{\Delta t}$$

$$\frac{\Delta m_1}{\Delta t} = \frac{\rho_1 \Delta V_1}{\Delta t} = \frac{\rho_1 A_1 \Delta l_1}{\Delta t} = \rho_1 A_1 v_1$$

$$\frac{\Delta m_1}{\Delta t} = \frac{\Delta m_2}{\Delta t}$$

(incompressible)

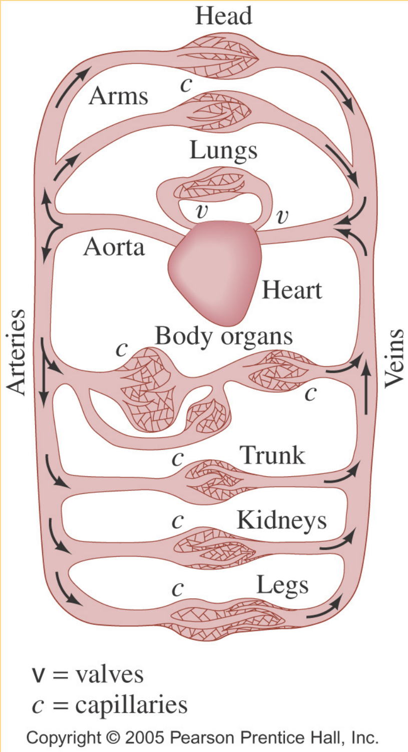
<u>**Equation of continuity:**</u> $\rho_1 A_1 v_1 = \rho_2 A_2 v_2 \Rightarrow \boxed{A_1 v_1 = A_2 v_2}$

$Av = A\Delta l / \Delta t = \Delta V / \Delta t \Rightarrow Av$ *is the* ***volume rate of flow***

Where the pipe is **wider** the flow is **slower**

A river flows slowly through a meadow where it is broader, but speeds up to torrential speed when passing through a narrow gorge.

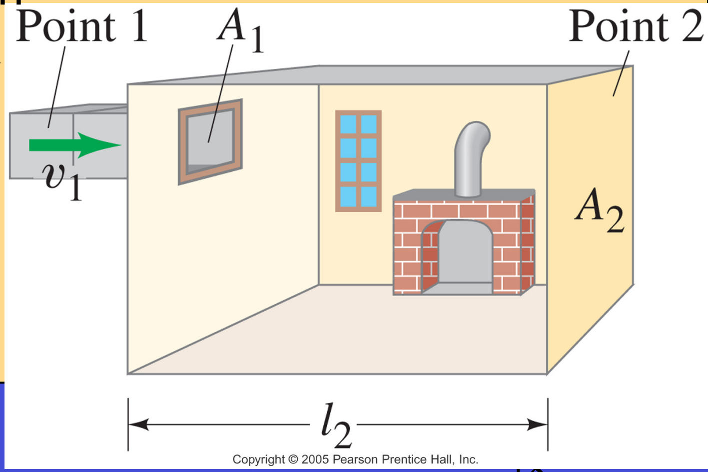
Exercises



Ex. 10-11 In humans, blood flows from the heart to the aorta, then to arteries and to tiny capillaries. It returns to the heart via the veins. The radius of the aorta is about 1.2 cm and the blood through it has a speed of 40 cm/s. A typical capillary has a radius of about 4x10^(-4) cm and blood flows through it at 5x10^(-4) m/s. Estimate the number of capillaries in the body.

$$A_2 v_2 = A_1 v_1 \Rightarrow v_2 N \pi r_{cap}^2 = v_1 \pi r_{aorta}^2 \Rightarrow N \approx 7 \times 10^9$$

Ex. 10-12 What area must a heating duct have if air moving 3.0 m/s along it can replenish the air every 15 minutes in a room of volume 300 m^3?



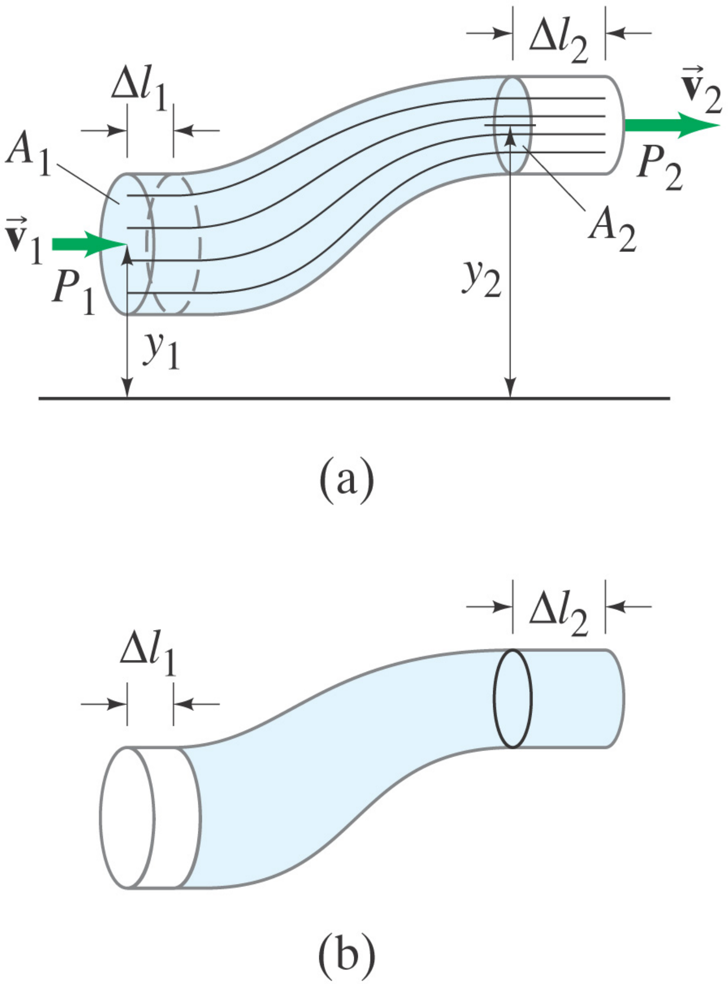
$$A_1 v_1 = A_2 v_2 = A_2 l_2 / t = V_2 / t$$

$$A_1 = \frac{V_2}{v_1 t} = 0.11 m^2$$

Bernoulli's Equation

Where the **velocity of a fluid is high**, the **pressure is low**

velocity of a fluid is low, the **pressure is high**



$$W = W_1 + W_2 + W_g = F_1\Delta l_1 - F_2\Delta l_2 - mg(y_2 - y_1)$$

$$W = P_1A_1\Delta l_1 - P_2A_2\Delta l_2 - mgy_2 + mgy_1$$

$$\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = P_1A_1\Delta l_1 - P_2A_2\Delta l_2 - mgy_2 + mgy_1$$

since $m = \rho A_1\Delta l_1 = \rho A_2\Delta l_2$

$$\frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 = P_1 - P_2 - \rho g y_2 + \rho g y_1$$

Bernoulli's equation:

$$\boxed{P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2}$$

Exercises and Applications

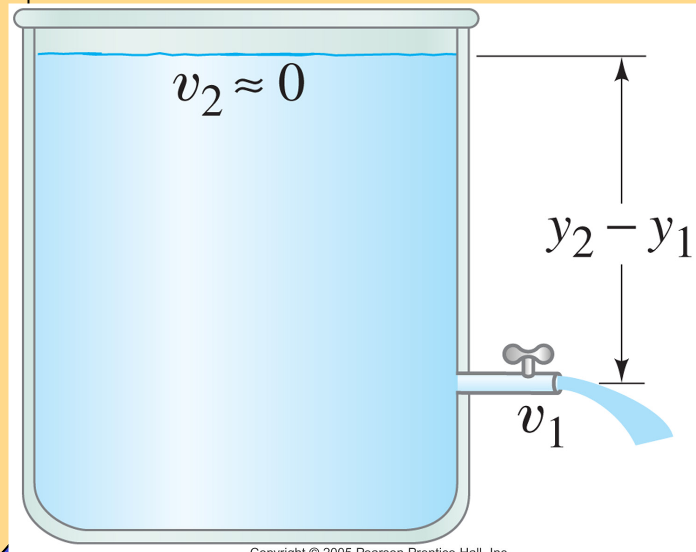
Ex. 10-13 Water circulates throughout a house in a hot-water heating system. If the water is pumped at a speed of 0.50 m/s through a 4.0-cm diameter pipe in the basement under a pressure of 3.0 atm, what will be the flow speed and pressure in a 2.6-cm-diameter pipe on the second floor 5.0 m above?

$$\boxed{A_1 v_1 = A_2 v_2}$$

$$\boxed{P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2}$$

$$A_1 v_1 = A_2 v_2 \Rightarrow v_2 = v_1 \frac{\pi r_1^2}{\pi r_2^2} = 1.2 m/s$$

$$P_2 = P_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) + \rho g(y_1 - y_2) = 2.5 atm$$



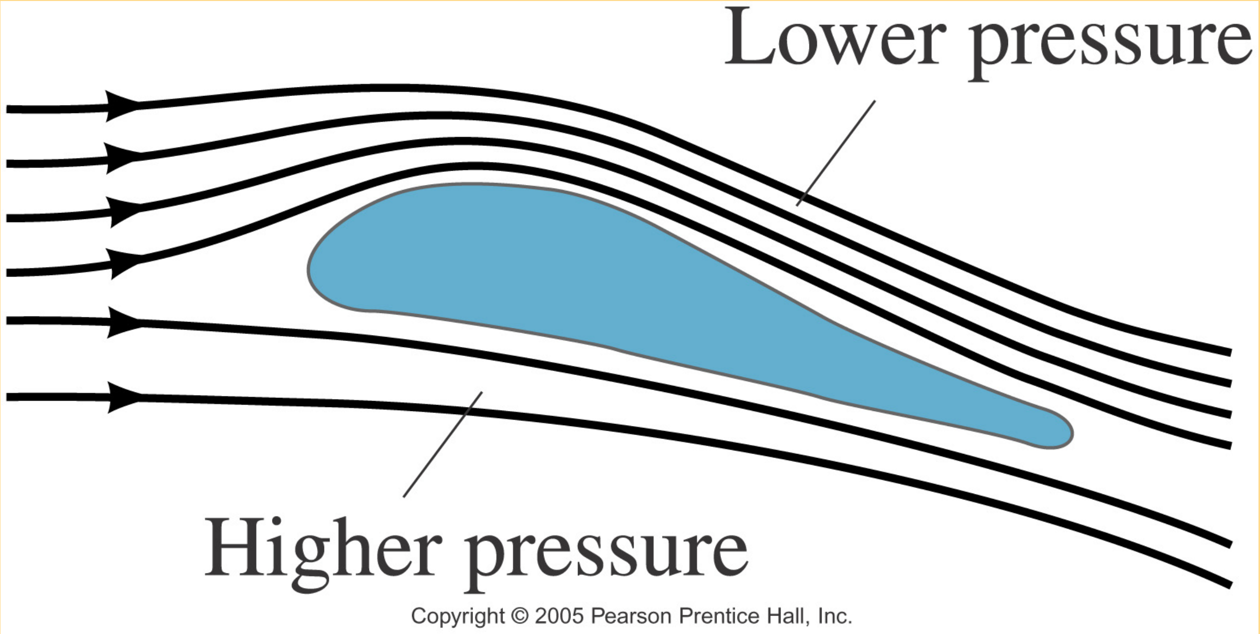
Torricelli's theorem.

$$P_1 = P_2 \Rightarrow v_1 = \sqrt{2g(y_2 - y_1)}$$

Liquid leaves the spigot with the same speed that a freely falling object would attain if falling from the same height.

Application - airplane

Lift on an airplane **wing** is due to the different air **speeds** and **pressures** on the two surfaces of the wing.



Surface Tension

The **surface** of a liquid at rest is not perfectly flat; it **curves** either up or down at the **walls** of the container. This is the result of **surface tension**, which makes the surface behave somewhat elastically.